

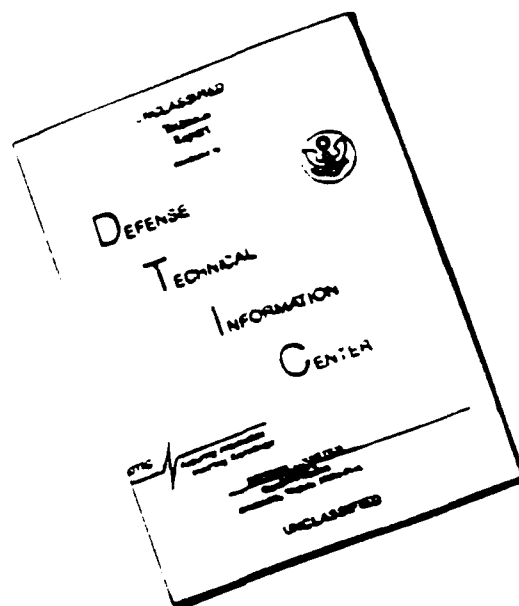


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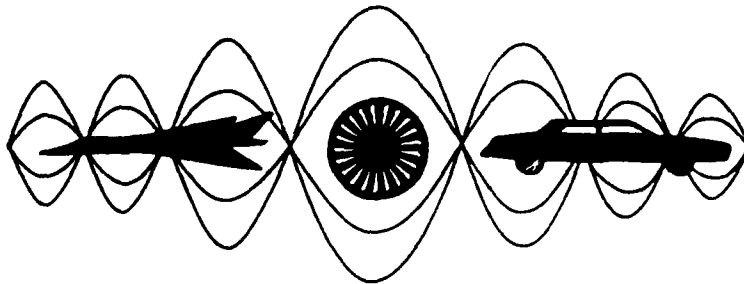
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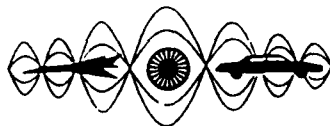
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A COMPARISON OF MEMBRANE, VACUUM, AND FLUID LOADED SPHERICAL SHELL MODELS  
WITH EXACT RESULTS

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ABSTRACT

A new set of spherical shell theories is presented with differing degrees of physicality, varying from a simple membrane model up to a fluid loaded shell theory that includes translational motion, rotary inertia, and transverse shearing-stress. Numerical results from these theories are compared with exact results from elastodynamic theory. The motivation of this study is to overcome the limitations of both modal analysis approaches and of the somewhat more general Extended Boundary Condition (EBC) method due to Waterman, sometimes called the T-matrix method. The spherical shell is studied with an eye towards generalizing the results obtained to spheroids. The aim of the present research is to yield a better understanding of the scattering event by employing more general and more physical shell theories.

INTRODUCTION

So-called "shell theories" simplify the calculations of the motion of thin elastic shells by making assumptions about the scatterer and its movements. We use the standard assumptions of shell theory as formulated by A. E. H. Love [1] and which are as follows: first the thickness of a shell is small compared with the smallest radius of curvature of the shell; second the displacement is small in comparison with the shell thickness; third the transverse normal stress acting on planes parallel to the shell middle surface is negligible; and finally the fibers of the shell normal to the middle surface remain so after deformation and are themselves not subject to elongation. These assumptions are used in the development of a shell theory for an elastic spherical shell in the spirit of Timoshenko-Mindlin[2,3] plate theory.

DERIVATION OF THE EQUATIONS OF MOTION

In spherical shells membrane stresses (proportional to  $\beta$ ) predominate over flexural stresses (proportional to  $\beta^2$ ) where

$$\beta = \frac{1}{\sqrt{12}} \frac{h}{a}. \quad (1)$$

We differ from the standard derivation for the sphere [4] by retaining all terms of order  $\beta^2$  in both the kinetic and potential energy parts of the Lagrangian and by considering the resonance frequencies for the fluid loaded case to be complex. We note that this level of approximation will allow us to include the effects of rotary inertia and shear distortion in our shell theory, as well as damping by fluid loading. The parameter  $\beta$  itself is proportional to the radius of gyration of a differential element of the shell and arises from integration through the thickness of the shell in a radial direction. We will use an implicit harmonic time variation of the form  $\exp(-i\omega t)$ . We begin our derivation by considering a u,v,w axis system on the middle surface of a spherical shell of radius  $a$  (measured to mid-shell) with thickness  $h$ , as shown in Fig. 1.

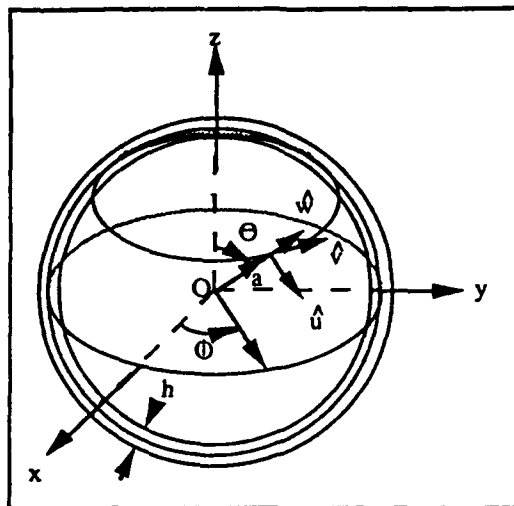


Fig. 1. --- Spherical shell showing coordinates used. The  $\hat{u}$ ,  $\hat{v}$ ,  $\hat{w}$  - coordinate system is somewhere on the mid-surface.

#### Lagrangian variational analysis

The Lagrangian,  $L$ , is

$$L = T - V + W, \quad (2)$$

where  $T$  is the kinetic energy,  $V$  is the potential energy, and  $W$  is the work due to the pressure at the surface. The kinetic energy is given by

$$T = \frac{1}{2} \rho_s \int_0^{2\pi} \int_0^\pi \int_{-h/2}^{h/2} (\dot{u}_s^2 + \dot{w}_s^2) (a+x)^2 \sin \theta dx d\theta d\phi, \quad (3)$$

where the surface displacements are taken to be linear as in Timoshenko-Mindlin plate theory:

$$\dot{u}_s = \left(1 + \frac{x}{a}\right) \dot{u} - \frac{x}{a} \frac{\partial \dot{w}}{\partial \theta}, \quad (4)$$

and

$$\dot{w}_s = \dot{w}. \quad (5)$$

The motion of the spherical shell is axisymmetric since the sound field is torsionless. Thus there is no motion in the  $v$ -direction. Substitution of Eqs. (4) and (5) into Eq. (3) yields, after integration over  $x$  and  $\theta$ ,

$$T = \pi \rho_s \int_0^\pi \sin \theta \left[ \left( \frac{h^5}{80a^2} + \frac{h^3}{2} + ha^2 \right) \dot{u}^2 - 2 \left( \frac{h^5}{80a^2} + \frac{h^3}{4} \right) \dot{u} \frac{\partial \dot{w}}{\partial \theta} + \left( \frac{h^5}{80a^2} + \frac{h^3}{12} \right) \left( \frac{\partial \dot{w}}{\partial \theta} \right)^2 + \left( \frac{h^3}{12} + ha^2 \right) \dot{w}^2 \right] d\theta, \quad (6)$$

or, in terms of  $\beta$ ,

$$T = \pi \rho_s ha^2 \int_0^\pi \left[ (1.8\beta^4 + 6\beta^2 + 1) \dot{u}^2 - (3.6\beta^4 + 6\beta^2) \dot{u} \frac{\partial \dot{w}}{\partial \theta} + (1.8\beta^4 + \beta^2) \left( \frac{\partial \dot{w}}{\partial \theta} \right)^2 + (\beta^2 + 1) \dot{w}^2 \right] \sin \theta d\theta, \quad (7)$$

where the first and last terms in square brackets in Eq. (7) are associated with linear translational kinetic energy and the middle two terms are associated with rotational kinetic energies of an element of the shell.

The potential energy of the shell is

$$V = \frac{1}{2} \int_0^{2\pi} \int_0^\pi \int_{-h/2}^{h/2} (\sigma_{\theta\theta} \epsilon_{\theta\theta} + \sigma_{\phi\phi} \epsilon_{\phi\phi}) (x+a)^2 \sin \theta dx d\theta d\phi,$$

where the nonvanishing components of the strain are

$$\epsilon_{\theta\theta} = \frac{1}{a} \left( \frac{\partial u}{\partial \theta} + w \right) + \frac{x}{a^2} \left( \frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2} \right), \quad (9)$$

and

$$\epsilon_{\phi\phi} = \frac{1}{a} (\cot \theta u + w) + \frac{x}{a^2} \cot \theta \left( u - \frac{\partial w}{\partial \theta} \right), \quad (10)$$

and where the nonzero stress components are

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2} (\epsilon_{\theta\theta} + \nu \epsilon_{\phi\phi}), \quad (11)$$

and

$$\sigma_{\phi\phi} = \frac{E}{1-\nu^2} (\epsilon_{\phi\phi} + \nu \epsilon_{\theta\theta}), \quad (12)$$

where  $E$  is Young's modulus. By substitution the potential energy becomes

$$V = \frac{1}{2} \int_0^\pi \int_0^{2\pi} \int_{-h/2}^{h/2} \left[ \frac{E}{1-\nu^2} \frac{1}{(x+a)^2} \left\{ \left[ \left( 1 + \frac{x}{a} \right) \frac{\partial u}{\partial \theta} - \frac{x}{a} \frac{\partial^2 w}{\partial \theta^2} + w^2 \right] + \left[ \cot \theta \left[ \left( 1 + \frac{x}{a} \right) u - \frac{x}{a} \frac{\partial w}{\partial \theta} \right] + w \right]^2 \right. \right. \\ \left. \left. + 2 \nu \left[ \cot \theta \left[ \left( 1 + \frac{x}{a} \right) u - \frac{x}{a} \frac{\partial w}{\partial \theta} \right] + w \right] \left[ \left( 1 + \frac{x}{a} \right) \frac{\partial u}{\partial \theta} - \frac{x}{a} \frac{\partial^2 w}{\partial \theta^2} + w \right] \right\} \right] (x+a)^2 \sin \theta dx d\theta d\phi, \quad (13)$$

which after integration is

$$V = \frac{\pi E h}{1-\nu^2} \int_0^\pi \left\{ \left( w + \frac{\partial u}{\partial \theta} \right)^2 + (w + u \cot \theta)^2 + 2 \nu \left( w + \frac{\partial u}{\partial \theta} \right) (w + u \cot \theta) \right. \\ \left. + \beta^2 \left[ \left( \frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2} \right)^2 \cot^2 \theta \left( u - \frac{\partial w}{\partial \theta} \right)^2 + 2 \nu \cot \theta \left( u - \frac{\partial w}{\partial \theta} \right) \left( \frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2} \right) \right] \right\} \sin \theta d\theta. \quad (14)$$

Terms in the potential energy proportional to  $\beta^2$  are due to bending stresses.

And finally, the work done by the pressure of the surrounding fluid on the spherical shell is given by

$$W = 2\pi a^2 \int_0^\pi p_s w \sin \theta d\theta, \quad (15)$$

where  $p_s$  is the pressure at the surface.

#### The Lagrangian density and its equations of motion

A Lagrangian density must be used instead of the Lagrangian since the integration along the polar angle is intrinsic to the problem. The Lagrangian density is

$$\begin{aligned} \mathcal{L} = & \pi \rho_s h a^2 [(1 + 6\beta^2 + 1.8\beta^4) \dot{u}^2 - (6\beta^2 + 3.6\beta^4) \dot{u} \frac{\partial \dot{w}}{\partial \theta} + (\beta^2 + 1.8\beta^4) (\frac{\partial \dot{w}}{\partial \theta})^2 \\ & + (1 + \beta^2) \dot{w}^2] \sin \theta - \frac{\pi E h}{1 - \nu^2} [(w + \frac{\partial u}{\partial \theta})^2 + (w + u \cot \theta)^2 + 2\nu(w + \frac{\partial u}{\partial \theta})(w + u \cot \theta) \\ & + 2\pi a^2 p_s w \sin \theta, \end{aligned} \quad (16)$$

with corresponding differential equations of motion

$$0 = \frac{\partial \mathcal{L}}{\partial u} - \frac{d}{d\theta} \frac{\partial \mathcal{L}}{\partial u_\theta} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{u}}, \quad (17)$$

and

$$0 = \frac{\partial \mathcal{L}}{\partial w} - \frac{d}{d\theta} \frac{\partial \mathcal{L}}{\partial w_\theta} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{w}} + \frac{d^2}{d\theta dt} \frac{\partial \mathcal{L}}{\partial w_{\theta t}} + \frac{d^2}{d\theta^2} \frac{\partial \mathcal{L}}{\partial w_{\theta\theta}}, \quad (18)$$

where subscripts denote differentiation of the variable with respect to the subscript.

By substitution of Eqs. (17) and (18) into (16) we obtain

$$\begin{aligned} 0 = & (1 + \beta^2) \left[ \frac{\partial^2 u}{\partial \theta^2} + \cot \theta \frac{\partial u}{\partial \theta} - (\nu + \cot^2 \theta) u \right] - \beta^2 \frac{\partial^3 w}{\partial \theta^3} - \beta^2 \cot \theta \frac{\partial^2 w}{\partial \theta^2} \\ & + [(1 + \nu) + \beta^2(\nu + \cot^2 \theta)] \frac{\partial w}{\partial \theta} - \frac{a^2}{c_p^2} [(1.8\beta^4 + 6\beta^2 + 1) \frac{\partial^2 u}{\partial t^2} - (1.8\beta^4 + 3\beta^2) \frac{\partial^3 w}{\partial \theta \partial t^2}], \end{aligned} \quad (19)$$

and

$$\begin{aligned} -p_s \frac{(1 - \nu^2)a^2}{Eh} = & \beta^2 \frac{\partial^3 u}{\partial \theta^3} + 2\beta^2 \cot \theta \frac{\partial^2 u}{\partial \theta^2} - [(1 + \nu)(1 + \beta^2) + \beta^2 \cot^2 \theta] \frac{\partial u}{\partial \theta} \\ & + \cot \theta [(2 - \nu + \cot^2 \theta)\beta^2 - (1 + \nu)] u - \beta^2 \frac{\partial^4 w}{\partial \theta^4} - 2\beta^2 \cot \theta \frac{\partial^3 w}{\partial \theta^3} \\ & + \beta^2 (1 + \nu + \cot^2 \theta) \frac{\partial^2 w}{\partial \theta^2} - \beta^2 \cot \theta (2 - \nu + \cot^2 \theta) \frac{\partial w}{\partial \theta} - 2(1 + \nu) w \\ & + \frac{a^2}{c_p^2} [-(1.8\beta^4 + 3\beta^2) \frac{\partial^3 u}{\partial \theta \partial t^2} - (1.8\beta^4 + 3\beta^2) \cot \theta \frac{\partial^2 u}{\partial t^2} \\ & + (1.8\beta^4 + \beta^2) \frac{\partial^4 w}{\partial \theta^2 \partial t^2} + (1.8\beta^4 + \beta^2) \frac{\partial^3 w}{\partial \theta \partial t^2} \cot \theta - (\beta^2 + 1) \frac{\partial^2 w}{\partial t^2}]. \end{aligned} \quad (20)$$

These differential equations of motion (19) and (20) have solutions of the form

$$u(\eta) = \sum_{n=0}^{\infty} U_n (1 - \eta^2)^{1/2} \frac{dP_n}{d\eta},$$

and

$$w(\eta) = \sum_{n=0}^{\infty} W_n P_n(\eta),$$

where  $\eta = \cos \theta$  and  $P_n(\eta)$  are the Legendre polynomials of the first kind of order  $n$ . When the differential equations of motion (19) and (20) are expanded in terms of Eqs. (21) and (22), we obtain a set of linear equations in terms of  $U_n$  and  $W_n$ , whose determinant must vanish. We shall consider two cases: with and without fluid loading.

#### The vacuum case

The vacuum case is the simpler problem that occurs when the spherical shell is surrounded by a vacuum such that there is no damping. Thus, the pressure at the surface vanishes:  $p_s = 0$ . The set of linear equations the expansion coefficients must satisfy are

$$0 = [\Omega^2(1 + 6\beta^2 + 1.8\beta^4) - (1 + \beta^2)\kappa]U_n + [\Omega^2(3\beta^2 + 1.8\beta^4) - \beta^2\kappa - (1 + \nu)]W_n, \quad (23)$$

and

$$0 = -\lambda_n[(\kappa - 3)\beta^2 - 1.8\beta^4 + 1 + \nu]U_n + [\Omega^2(1 + 2\beta^2 + 1.8\beta^4) - 2(1 + \nu) - \beta^2\kappa\lambda_n]W_n, \quad (24)$$

where  $\Omega = \omega a / c_p$ ,  $\kappa = \nu + \lambda_n - 1$ , and  $\lambda_n = n(n+1)$ . In order for Eqs. (23) and (24) to be satisfied simultaneously with a non-trivial solution the determinant of the system must vanish:

$$\begin{aligned} 0 = & \Omega^4(1 + 6\beta^2 + 1.8\beta^4)(1 + 2\beta^2 + 1.8\beta^4) + \Omega^2\{(3\beta^2 + 1.8\beta^4)\lambda_n[(\kappa - 3)\beta^2 - 1.8\beta^4 + 1 + \nu] \\ & - [2(1 + \nu) + \beta^2\kappa\lambda_n](1 + 6\beta^2 + 1.8\beta^4) - (1 + \beta^2)\kappa(1 + 2\beta^2 + 1.8\beta^4)\} \\ & + (1 + \beta^2)\kappa[2(1 + \nu) + \beta^2\kappa\lambda_n] - \lambda_n[(\kappa - 3)\beta^2 - 1.8\beta^4 + 1 + \nu](\beta^2 + 1 + \nu). \end{aligned} \quad (25)$$

Since there are no damping terms, the shell vibrates theoretically forever. Thus, the normalized frequency  $\Omega$  can be taken to be real. Equation (25) is quadratic in  $\Omega^2$ , thus we expect two real roots to (25) and thus two modes for the motion of the shell. They are the symmetric and antisymmetric Lamb modes.

#### The fluid loaded case

The fluid loaded case requires that we consider a modal expansion of the surface pressure in terms of the specific acoustic impedance  $z_n$ . In its most general form this is

$$p(a, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} z_n W_{nm} P_n^m(\cos \theta) \cos m\phi, \quad (26)$$

where

$$z_n = i\rho c \frac{h_n(ka)}{h_n'(ka)}. \quad (27)$$

The specific acoustic impedance  $z_n$  can be split into real and imaginary parts:

$$z_n = r_n - i\omega m_n, \quad (28)$$

where

$$r_n = \rho c \operatorname{Re} \left\{ \frac{ih_n(ka)}{h_n'(ka)} \right\}, \quad (29)$$



and

$$m_n = -\frac{\rho c}{\omega} \operatorname{Im} \left\{ \frac{i h_n(ka)}{h'_n(ka)} \right\}. \quad (30)$$

For the case of axisymmetric motion we are considering, the surface pressure is

$$p_s(\theta) = -\sum_{n=0}^{\infty} z_n \dot{W}_n P_n(\cos \theta), \quad (31)$$

or by substitution,

$$p_s(\theta) = -\sum_{n=0}^{\infty} (-i\omega W_n r_n - \omega^2 W_n m_n) P_n(\cos \theta). \quad (32)$$

Use of Eq. (32) in our set of differential equations of motion (19) and (20) yields the following set of linear equations for the expansion coefficients in the case of a fluid loaded spherical shell:

$$0 = [\Omega^2(1 + 6\beta^2 + 1.8\beta^4) - (1 + \beta^2)\kappa]U_n + [\Omega^2(3\beta^2 + 1.8\beta^4) - \beta^2\kappa - (1 + \nu)]W_n, \quad (33)$$

and

$$0 = -\lambda_n[(\kappa - 3)\beta^2 - 1.8\beta^4 + 1 + \nu]U_n + [\Omega^2(1 + \alpha + 2\beta^2 + 1.8\beta^4) - 2(1 + \nu) + \Omega i\gamma - \beta^2\kappa\lambda_n]W_n, \quad (34)$$

where

$$\alpha = \frac{m_n}{\rho_s h}, \quad (35)$$

and

$$\gamma = \frac{a}{h} \frac{r_n}{\rho_s c_p}. \quad (36)$$

Again the determinant of Eqs. (33) and (34) must vanish. However, in this instance the value of  $\Omega$  must be taken to be complex; the resonances have a width that depends on the damping. The result of setting this determinant to zero is

$$\begin{aligned} 0 = & \Omega^4(1 + 6\beta^2 + 1.8\beta^4)(1 + \alpha + 2\beta^2 + 1.8\beta^4) \\ & + \Omega^3 i\gamma(1 + 6\beta^2 + 1.8\beta^4) + \Omega^2 \{ (3\beta^2 + 1.8\beta^4)\lambda_n [(\kappa - 3)\beta^2 - 1.8\beta^4 + 1 + \nu] \\ & - [2(1 + \nu) + \beta^2\kappa\lambda_n](1 + 6\beta^2 + 1.8\beta^4) - (1 + \beta^2)\kappa(1 + \alpha + 2\beta^2 + 1.8\beta^4) \} + \Omega \{ -i\gamma(1 + \beta^2)\kappa \\ & + (1 + \beta^2)\kappa[2(1 + \nu) + \beta^2\kappa\lambda_n] - \lambda_n[(\kappa - 3)\beta^2 - 1.8\beta^4 + 1 + \nu](\beta^2 + 1 + \nu) \}. \end{aligned}$$

Equation (37) has at least four complex roots. From work with an exact modal solution to the problem we expect two roots to be associated with the symmetric and antisymmetric modes of the shell. We expect the other two roots to be associated with a water-borne pseudo-Stoneley wave.

## CONCLUSIONS

The next step is to plot the roots of Eqs. (25) and (37) to compare the resonances predicted by these models with those given by exact modal expansion solutions. By suppressing  $\alpha$  and  $\gamma$ , the model associated with Eq. (37) reverts to the vacuum case model associated with Eq. (25). Similarly suppression of factors of  $\beta$  in Eq. (25) will result in a reversion to a previously derived solution (Junger and Feit, 1986). We may then rank the different models according to their degree of physicality and compare their results for various relative thicknesses against each other and against the exact results of the modal expansion model. We may also compare the limitations of each of the models including the exact solution, as well as those of shell models in general.

By setting the values of  $\alpha$  and  $\gamma$  in Eq. (37) to zero, we revert the shell theory model to one without fluid loading. Similarly, by setting  $\beta$  to zero as well, the model reverts to a membrane model. These models, fluid loaded, vacuo case, and membrane, are successively less physically sophisticated and give successively less good comparison with exact (modal expansion) results. Starting with the least sophisticated model, we see in Fig. 2 thick spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by the membrane model. Here and in the succeeding figures thick means  $h/a = 0.1$ ; thin means  $h/a = 0.01$ . The shell material is a generic steel with density  $\rho_s = 7.7$  times that of water, shear velocity  $v_s = 3.24$  km/s, and longitudinal velocity  $v_l = 5.95$  km/s. The surrounding fluid is taken to be water with density  $\rho = 1000$  kg/m<sup>3</sup> and sound velocity  $c_s = 1.4825$  km/s. The symmetric mode shows a good comparison between exact and shell theory predictions, but the antisymmetric shell theory results for this approximation compare poorly with the exact flexural results. Note that some symmetric mode resonances were not found by our exact theory algorithm. In Fig. 3 we see thin spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by the membrane model. Again there is good comparison between dilatational (symmetric) mode resonances calculated by the two methods, except for the first couple of resonances. Only a few exact flexural resonances were picked up by our algorithm. And again the shell theory flexural (antisymmetric) mode resonances show they do not asymptote properly with increasing order. In Fig. 4 we have thick spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory without fluid loading (vacuum). As in the membrane model the shell theory and exact calculations compare well for the dilatational (symmetric) mode resonances. In contrast with the membrane model, however, the exact and shell theory calculations for this model show much better agreement for the flexural (antisymmetric) mode resonances. This model does not include fluid loading, but does include the effects of shear distortion and rotary inertia. The vacuum shell theory flexural mode resonances do not asymptote for large size parameter  $ka$  to the exact results, however. In Fig. 5 we see thin spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory without fluid loading (vacuum). As in the membrane model the shell theory and exact calculations compare well for the dilatational (symmetric) mode resonances except for the first couple of resonances. This vacuum model does not have fluid loading, and has insufficient damping for the first two dilatational (symmetric) mode resonances. Again, the flexural (symmetric) mode resonances show roughly the correct behavior, but it is not possible to tell what the asymptotic value of the phase velocity would be for large size parameter on this scale. Next in Fig. 6 we have a plot of thick spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory with fluid loading. As in the vacuum case as well as for the membrane model, the dilatational (symmetric) mode resonances compare well for exact and shell theory methods. The flexural (antisymmetric) mode resonances, as calculated by shell theory with fluid loading, do not appear to have the correct asymptotic limit for large size parameter, although they do exhibit roughly the correct behavior for lower values of  $ka$ . Finally, in Fig. 7 we see thin spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory with fluid loading. The exact and shell theory calculations agree well for the dilatational (symmetric) resonances and exhibit a marked improvement for the first several shell theory symmetric mode resonances. This is due to the inclusion of fluid loading in the model. The flexural (antisymmetric) mode resonances show the appropriate behavior on this rather limited size parameter scale.

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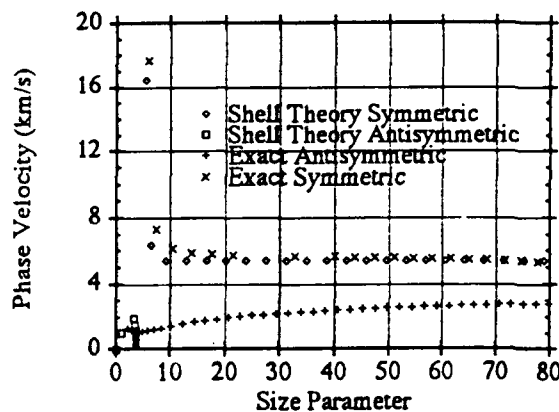


Fig. 2. Thick steel shell, membrane model ( $h/a = 0.1$ ). Fig. 3. Thin steel shell, membrane model ( $h/a = 0.01$ ).

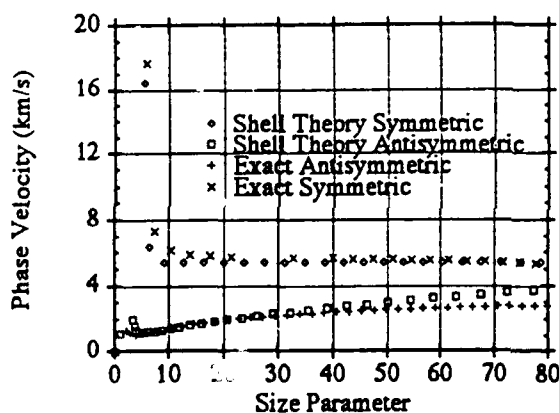
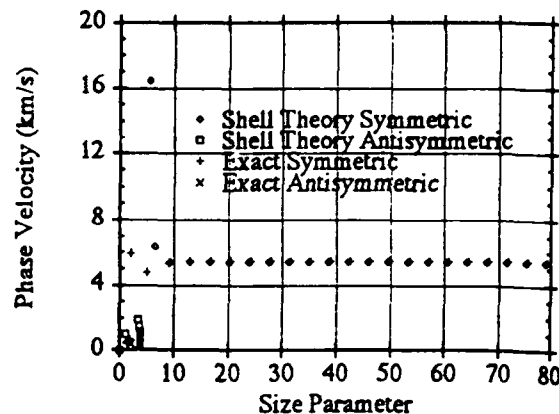


Fig. 4. Thick steel shell, vacuum model ( $h/a = 0.1$ ).

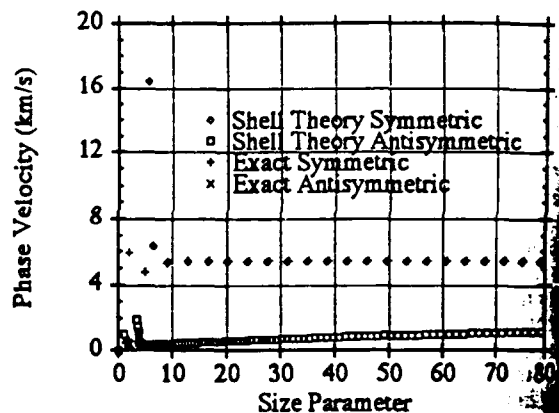


Fig. 5. Thin steel shell, vacuum model ( $h/a = 0.01$ ).

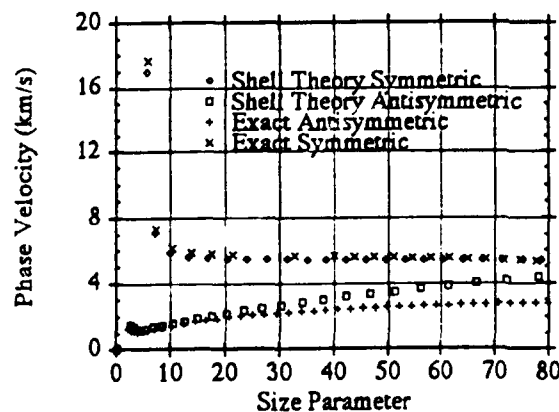


Fig. 6. Thick steel shell, fluid loaded ( $h/a = 0.1$ ).

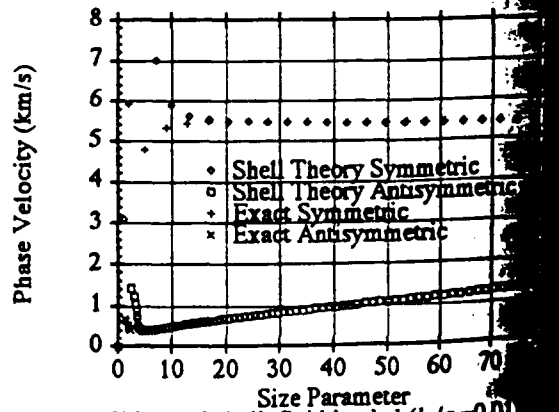


Fig. 7. Thin steel shell, fluid loaded ( $h/a = 0.01$ ).